Advanced Mathematics for Business
Topic 6: Differentiation 2
Introductory Management Statistics

Scope and Coverage

This topic will cover:

- Partial differentiation
- Total differential

Learning Objectives

By the end of this topic students will be able to:

- Carry out partial differentiation
- Relate partial differentiation to optimisation
- Calculate partial point elasticities
- Recognise the total differential

Introductory Exercise

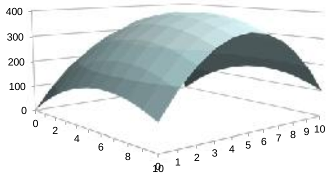
Differentiate *y* with respect to *S in each of the following*:

= −4 + 50	— = −8 + 50
= (2 + 5)	— = 6(2 + 5)
= e	— = ae
= a	— = a a
= .	— = . (1 + ( ))

Multivariate Function

- Functions with more than one independent variable
  - $y = f(x_1, x_2, x_3 \ldots)$
  - For example

$z = -4x^2 - 10y^2 + 50x + 100y$



Definition and Notation

- Partial differentiation
  - Differentiate with respect to one variable, assuming all others are constant
- Notation

— rate of change of *z* with *x*, other variables assumed constant

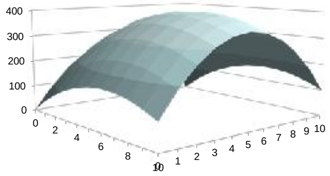
— rate of change of *z* with *y*, other variables assumed constant

Worked Example 1

- $z = -4x^2 - 10y^2 + 50x + 100y$

— = −8 + 50

— = −20 + 100



Worked Example 2

- Find partial derivatives with respect to x and y.

= 8 + 4 + — = 16 + — = 12 +

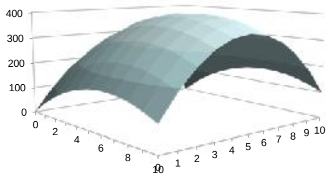
= — = 2 — = 3

Worked Example 3: Optimisation

A company sells two products and has developed a model of its annual profits based upon the product prices.

How should the prices be set?

= −4 − 10 + 50 + 100

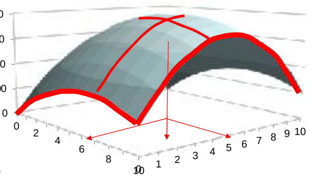


Worked Example 3: Optimisation

$= -4 \quad - 10 \quad + 50 \quad + 100$

$\frac{}{} = -8 \quad + 50 = 0$

$\frac{}{} = -20 \quad + 100 = 0$



$-8 \quad + 50 = 0 \Rightarrow \quad = 6.25$

$-20 \quad + 100 \Rightarrow \quad = 5$

- Hence maximum profit is 406.25

Elasticity of Demand – Theory

- Elasticity of demand

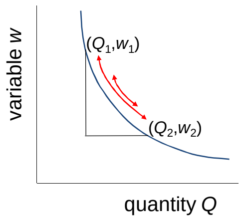
$= \frac{\%\text{ change in}}{\%\text{ change in}} = \frac{\Delta}{\Delta} \frac{}{}$

- Arc price elasticity of demand

$= \frac{(\quad - \quad)}{\frac{(\quad + \quad)}{2}} \frac{\frac{(\quad + \quad)}{2}}{(\quad - \quad)}$

- Point elasticity of demand

$= \lim_{\to} \frac{\Delta}{\Delta} \frac{}{} = \frac{}{}\frac{}{}$



Elasticity of Demand

- Own price elasticity of demand

$= \frac{}{} \frac{}{}$

- Cross price elasticity of demand

$= \frac{}{} \frac{}{}$

- Income elasticity of demand

$= \frac{}{} \frac{}{}$

Worked Example

Following research the demand for Brand 1 is thought to be characterised by;

= 50 − 2 + + 3 + 0.01 + other terms

What is the responsiveness of the quantity demanded to changes in own price, other brands' prices and income?

= ——, = ——, = ——, = ——

Worked Example

= 50 − 2 + + 3 + 0.01 + other terms

= —— = −2 —

= —— = —

= —— = 3 —

= —— = 0.01 —

Worked Example

= 100 − 4 + 2

What are the own price and cross price point elasticities at p_1 = £15, p_2 = £10?

= —— = −4 — = −1

= —— = 2 — = 0.33

Total Differential

= (+ + +...)

= — + — + — +...

Δ ≈ —Δ + —Δ + —Δ +...

Exercise

Show that for the demand function $Q_1=Q_1(p_1,p_2)$ that

$$\begin{bmatrix}\text{\% increase}\\ \text{in demand}\\ \text{for Brand 1}\end{bmatrix} = \begin{bmatrix}\text{own price}\\ \text{elasticity of}\\ \text{demand}\end{bmatrix} \times \begin{bmatrix}\text{\% increase}\\ \text{in price of}\\ \text{Brand 1}\end{bmatrix} + \begin{bmatrix}\text{cross price}\\ \text{elasticity of}\\ \text{demand}\end{bmatrix} \times \begin{bmatrix}\text{\% increase}\\ \text{in price of}\\ \text{Brand 2}\end{bmatrix}$$

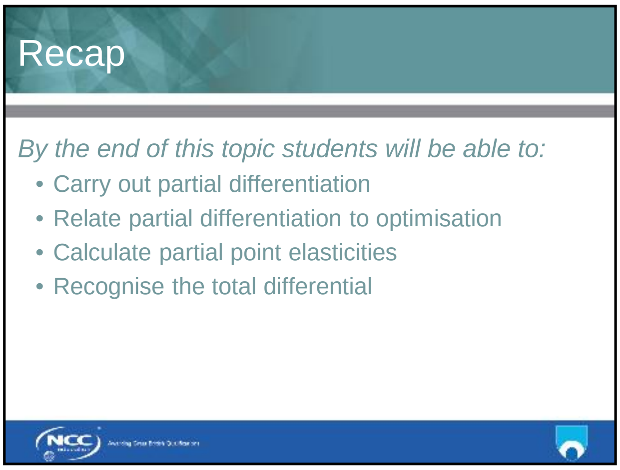
Exercise

Δ ≈ —Δ + —Δ

Δ ≈ Δ/— + Δ/—

Δ/— ≈ × Δ/— + × Δ/—

$$\begin{bmatrix}\text{\% increase}\\ \text{in demand}\\ \text{for Brand 1}\end{bmatrix} = \begin{bmatrix}\text{own price}\\ \text{elasticity of}\\ \text{demand}\end{bmatrix} \times \begin{bmatrix}\text{\% increase}\\ \text{in price of}\\ \text{Brand 1}\end{bmatrix} + \begin{bmatrix}\text{cross price}\\ \text{elasticity of}\\ \text{demand}\end{bmatrix} \times \begin{bmatrix}\text{\% increase}\\ \text{in price of}\\ \text{Brand 2}\end{bmatrix}$$


Recap
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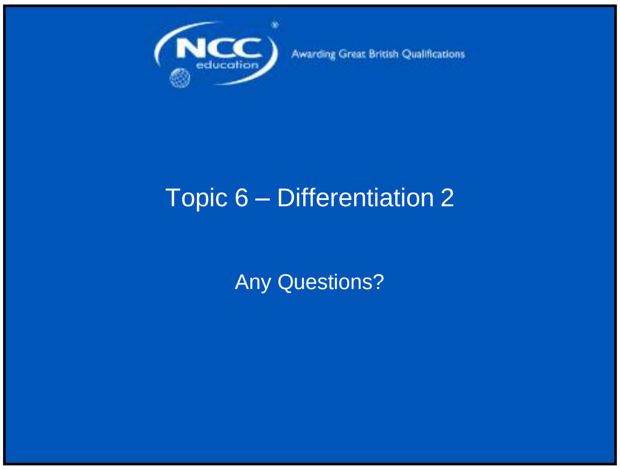
Awarding Great British Qualifications
Topic 6 – Differentiation 2
Any Questions?